

Digs Savings Account Analysis
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9 April 2019

Summary:

The Digs savings account (see <https://digs.co/>) has a unique earning schedule compared to a traditional savings account. A first analysis is performed to determine an equivalent APY for the Digs savings account, so it can be compared to the APY of a traditional savings account. A second analysis is performed to determine if contributing to a Digs savings account is profitable from a mortgage rate perspective.

Equivalent APY Analysis:

The equation for compounding interest is shown below:

$$A = P \left(1 + \frac{r}{n} \right)^{nt} \quad (1)$$

where:

A=total amount after compounding

P=principal before compounding

r=interest rate

n=number of times interest is compounded per year

t=time in years

In this analysis, gain will be defined as the ratio of amount after compounding to principal before compounding. The equation for gross gain is shown below:

$$G = \frac{A}{P} \quad (2)$$

In a traditional savings account, interest is considered taxable income. The equation for after-tax interest is shown below:

$$I = (1 - t)(A - P) \quad (3)$$

where:

t=tax bracket as a proportion (0.1 for 10% tax bracket, 0.12 for 12% tax bracket, etc.)

Accounting for tax, Equations (2) and (3) can be combined to determine after-tax gain:

$$G = \frac{I + P}{P} = \frac{(1 - t)(A - P) + P}{P} = \frac{(1 - t)(A - P)}{P} + 1 \quad (4)$$

In a traditional savings account, interest is compounded off of principal each month. With the Digs savings account, interest is paid in the form of a match contribution per month, similar to a 401(k) matching plan (see <https://help.digs.co/banking/are-there-limits-to-digs-contributions>). Additionally, the match that the user earns is tax-free (see <https://help.digs.co/banking/how-do-digs-contributions-impact-my-taxes-each-year>). Calculated gains per tier are shown below for the Digs savings account:

$$G_{digs} = \begin{cases} 1.18 & \text{for tier 1*} \\ 1.1 & \text{for tier 2} \\ 1.05 & \text{for tier 3} \\ 1.01 & \text{for tier 4} \end{cases} \quad (5)$$

*Taking into account \$1/month cost of Digs savings account

Since the Digs savings account matches a percentage of each monthly deposit, the comparison to a traditional savings account will follow an equivalent deposit schedule. As an example, \$50 is deposited into the the Digs savings account every month, and each deposit receives a \$10 match. Since the alternative would be depositing \$50 into a traditional savings account each month, interest will be calculated on \$50 principal in the first month, on \$100 principal in the second month, and so on. With this staggered deposit approach into a traditional savings account, and assuming that the account compounds monthly ($n=12$), Equations (1) and (2) can be used to derive the gross gain for this investing schedule:

$$1 \text{ month: } A_1 = P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow G_1 = \frac{A_1}{P} = \left(1 + \frac{r}{12}\right)$$

$$2 \text{ months: } A_2 = P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow G_2 = \frac{A_2}{2P} = \frac{1}{2} \left[\left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) \right]$$

$$3 \text{ months: } A_3 = P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{3}{12})} + P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$$

$$G_3 = \frac{A_3}{3P} = \frac{1}{3} \left[\left(1 + \frac{r}{12}\right)^3 + \left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) \right]$$

Using the summation symbol, the general equation for gross gain in a traditional savings account with this investing schedule for m months can be derived:

$$G_m = \frac{1}{m} \sum_{i=1}^m \left(1 + \frac{r}{12}\right)^i \quad (6a)$$

where:

m =number of months until home purchase

However, Equation (6a) assumes tax-free growth. Equations (1) and (4) can be used to derive the after-tax gain for this investing schedule for total investing time of 1 month to 12 months inclusive:

$$1 \text{ month: } A_1 = P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow G_1 = \frac{(1-t)(A_1 - P)}{P} + 1 = (1-t) \left[\left(1 + \frac{r}{12}\right) - 1 \right] + 1$$

$$2 \text{ months: } A_2 = P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P\left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$$

$$G_2 = \frac{(1-t)(A_2 - 2P)}{2P} + 1 = \frac{(1-t)}{2} \left[\left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) - 2 \right] + 1$$

3 months: $A_3 = P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{3}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$

$$G_3 = \frac{(1-t)(A_3 - 3P)}{3P} + 1 = \frac{(1-t)}{3} \left[\left(1 + \frac{r}{12}\right)^3 + \left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) - 3 \right] + 1$$

Using the summation symbol, the general equation for after-tax gain in a traditional savings account with this investing schedule for m months can be derived:

$$G_m = (1-t) \left(\frac{\sum_{i=1}^m \left(1 + \frac{r}{12}\right)^i}{m} - 1 \right) + 1 \quad (6b)$$

$1 \leq m \leq 12$

Since tax is only applied once per year and contributions are occurring monthly, Equation (6a) only holds true if the total investing time is 12 months or less. The process can be repeated for a total investing time between 13 and 24 months inclusive:

13 months: $A_{13} = 12PG_{12} \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$

$$G_{13} = \frac{(1-t)(A_{13} - 13P)}{13P} + 1 = \frac{(1-t)}{13} \left[12G_{12} \left(1 + \frac{r}{12}\right) + \left(1 + \frac{r}{12}\right) - 13 \right] + 1$$

14 months: $A_{14} = 12PG_{12} \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$

$$G_{14} = \frac{(1-t)(A_{14} - 14P)}{14P} + 1 = \frac{(1-t)}{14} \left[12G_{12} \left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) - 14 \right] + 1$$

15 months: $A_{15} = 12PG_{12} \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{3}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{3}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{2}{12})} + P \left(1 + \frac{r}{12}\right)^{(12 \cdot \frac{1}{12})} \rightarrow$

$$G_{15} = \frac{(1-t)(A_{15} - 15P)}{15P} + 1 = \frac{(1-t)}{15} \left[12G_{12} \left(1 + \frac{r}{12}\right)^3 + \left(1 + \frac{r}{12}\right)^3 + \left(1 + \frac{r}{12}\right)^2 + \left(1 + \frac{r}{12}\right) - 15 \right] + 1$$

Using the summation symbol, the general equation for after-tax gain in a traditional savings account with this investing schedule for m months can be derived:

$$G_m = (1-t) \left(\frac{xG_x \left(1 + \frac{r}{12}\right)^{m-x} + \sum_{i=1}^{m-x} \left(1 + \frac{r}{12}\right)^i}{m} - 1 \right) + 1 \quad (6c)$$

$$x = 12 \text{ floor} \left(\frac{m-1}{12} \right)$$

where:

$G_0=0$

floor=flooring function (truncation)

Equation (6c) can be reduced to Equation (6b) by setting m to a value between 1 and 12 inclusive.

Equation (6c) can be reduced to Equation (6a) by setting t to 0.

Setting each Digs tier in Equation (5) equal to Equation (6c), the equivalent rate r_{eq} can be determined for any combinations of m and t.

Once r_{eq} is determined, this rate can be converted into an equivalent APY using the equation below:

$$APY_{eq} = 100 \left(\left(\frac{r_{eq}}{12} + 1 \right)^{12} - 1 \right) \quad (7)$$

where:

APY_{eq} =equivalent APY of Digs savings account

Match contribution earnings in the Digs savings account expire after five years (see <https://help.digs.co/banking/do-digs-contributions-ever-expire>). Therefore, equivalent APY values have been calculated for all combinations of m from 1 to 60 months, for t values according to the 2019 tax brackets, and for all four values of G_{digs} . Using the results on the following pages, optimal Digs savings account contribution amounts can be determined.

For each G_{digs} table, find the equivalent APY corresponding to the correct m and tax bracket. If this equivalent APY is greater than the APY of a traditional savings account, it is advantageous to contribute to this tier of the Digs savings account.

Gdig = 1.18		10%	12%	22%	24%	32%	35%	37%
m								
1		791.61%	832.99%	1108.15%	1181.65%	1574.46%	1779.22%	1940.49%
2		323.80%	336.56%	416.75%	437.03%	539.07%	588.81%	626.62%
3		193.82%	200.36%	240.42%	250.28%	298.49%	321.24%	338.24%
4		136.26%	140.43%	165.60%	171.69%	201.01%	214.57%	224.61%
5		104.44%	107.43%	125.30%	129.58%	149.95%	159.25%	166.10%
6		84.43%	86.74%	100.40%	103.65%	118.99%	125.94%	131.02%
7		70.75%	72.62%	83.59%	86.19%	98.38%	103.86%	107.86%
8		60.84%	62.39%	71.53%	73.68%	83.73%	88.22%	91.49%
9		53.33%	54.66%	62.46%	64.29%	72.81%	76.60%	79.36%
10		47.46%	48.62%	55.41%	57.00%	64.37%	67.65%	70.02%
11		42.74%	43.77%	49.77%	51.18%	57.67%	60.54%	62.62%
12		38.87%	39.80%	45.17%	46.42%	52.21%	54.77%	56.61%
13		39.17%	40.91%	51.68%	54.36%	67.50%	73.72%	78.38%
14		35.71%	37.18%	46.17%	48.38%	59.06%	64.04%	67.73%
15		32.82%	34.09%	41.76%	43.62%	52.54%	56.64%	59.68%
16		30.37%	31.48%	38.14%	39.74%	47.35%	50.82%	53.37%
17		28.27%	29.26%	35.12%	36.52%	43.13%	46.12%	48.31%
18		26.45%	27.33%	32.56%	33.80%	39.62%	42.25%	44.16%
19		24.85%	25.65%	30.35%	31.47%	36.67%	39.00%	40.69%
20		23.44%	24.17%	28.44%	29.45%	34.14%	36.23%	37.75%
21		22.18%	22.85%	26.76%	27.68%	31.95%	33.85%	35.22%
22		21.05%	21.67%	25.28%	26.12%	30.04%	31.77%	33.02%
23		20.04%	20.61%	23.96%	24.74%	28.35%	29.94%	31.09%
24		19.12%	19.65%	22.77%	23.49%	26.84%	28.32%	29.39%
25		20.09%	21.05%	27.04%	28.53%	35.79%	39.21%	41.76%
26		19.11%	19.99%	25.39%	26.71%	33.14%	36.14%	38.36%
27		18.23%	19.03%	23.93%	25.13%	30.87%	33.52%	35.48%
28		17.43%	18.17%	22.64%	23.72%	28.90%	31.27%	33.02%
29		16.69%	17.38%	21.49%	22.48%	27.18%	29.31%	30.88%
30		16.02%	16.66%	20.45%	21.36%	25.65%	27.60%	29.02%
31		15.41%	16.00%	19.52%	20.36%	24.30%	26.08%	27.38%
32		14.84%	15.39%	18.67%	19.45%	23.09%	24.73%	25.92%
33		14.31%	14.83%	17.90%	18.62%	22.01%	23.52%	24.62%
34		13.82%	14.31%	17.19%	17.86%	21.02%	22.43%	23.45%
35		13.36%	13.83%	16.53%	17.17%	20.13%	21.44%	22.39%
36		12.94%	13.38%	15.93%	16.53%	19.31%	20.54%	21.43%
37		13.77%	14.51%	19.13%	20.27%	25.92%	28.59%	30.57%
38		13.30%	13.99%	18.25%	19.31%	24.45%	26.86%	28.65%
39		12.86%	13.50%	17.46%	18.44%	23.15%	25.33%	26.95%
40		12.44%	13.05%	16.74%	17.64%	21.97%	23.97%	25.44%
41		12.06%	12.63%	16.08%	16.92%	20.92%	22.76%	24.10%
42		11.70%	12.23%	15.47%	16.25%	19.97%	21.66%	22.90%
43		11.36%	11.86%	14.91%	15.64%	19.10%	20.67%	21.82%
44		11.04%	11.52%	14.39%	15.08%	18.31%	19.77%	20.84%
45		10.74%	11.19%	13.91%	14.55%	17.59%	18.95%	19.95%
46		10.45%	10.89%	13.46%	14.07%	16.92%	18.20%	19.13%
47		10.18%	10.60%	13.04%	13.62%	16.31%	17.51%	18.38%
48		9.93%	10.32%	12.65%	13.19%	15.74%	16.88%	17.70%
49		10.64%	11.27%	15.25%	16.25%	21.18%	23.51%	25.25%
50		10.34%	10.94%	14.68%	15.60%	20.16%	22.30%	23.89%
51		10.07%	10.64%	14.14%	15.01%	19.23%	21.20%	22.66%
52		9.81%	10.35%	13.65%	14.47%	18.39%	20.21%	21.56%
53		9.56%	10.07%	13.20%	13.96%	17.63%	19.31%	20.56%
54		9.33%	9.82%	12.77%	13.49%	16.92%	18.49%	19.65%
55		9.11%	9.57%	12.37%	13.05%	16.27%	17.74%	18.82%
56		8.90%	9.34%	12.00%	12.64%	15.68%	17.05%	18.06%
57		8.70%	9.12%	11.65%	12.26%	15.12%	16.41%	17.36%
58		8.50%	8.91%	11.32%	11.90%	14.61%	15.83%	16.71%
59		8.32%	8.71%	11.02%	11.56%	14.13%	15.28%	16.12%
60		8.15%	8.52%	10.72%	11.25%	13.69%	14.77%	15.56%

Gdig = 1.10							
m	10%	12%	22%	24%	32%	35%	37%
1	254.07%	263.85%	325.26%	340.78%	418.83%	456.90%	485.85%
2	131.18%	135.36%	160.82%	167.04%	197.24%	211.38%	221.91%
3	87.14%	89.67%	104.75%	108.38%	125.69%	133.64%	139.49%
4	64.96%	66.73%	77.22%	79.71%	91.51%	96.86%	100.78%
5	51.68%	53.03%	61.00%	62.88%	71.72%	75.70%	78.60%
6	42.88%	43.97%	50.35%	51.86%	58.88%	62.02%	64.31%
7	36.62%	37.53%	42.84%	44.09%	49.90%	52.48%	54.36%
8	31.95%	32.73%	37.27%	38.34%	43.27%	45.47%	47.06%
9	28.33%	29.01%	32.98%	33.90%	38.19%	40.09%	41.47%
10	25.44%	26.05%	29.56%	30.38%	34.17%	35.85%	37.06%
11	23.09%	23.63%	26.79%	27.52%	30.91%	32.41%	33.49%
12	21.13%	21.63%	24.49%	25.15%	28.22%	29.57%	30.55%
13	21.37%	22.30%	28.03%	29.45%	36.36%	39.62%	42.04%
14	19.57%	20.37%	25.18%	26.36%	32.02%	34.64%	36.58%
15	18.06%	18.75%	22.89%	23.89%	28.65%	30.83%	32.43%
16	16.78%	17.38%	21.00%	21.86%	25.96%	27.82%	29.18%
17	15.67%	16.21%	19.41%	20.17%	23.76%	25.37%	26.55%
18	14.70%	15.19%	18.06%	18.74%	21.92%	23.35%	24.38%
19	13.85%	14.29%	16.89%	17.50%	20.36%	21.64%	22.56%
20	13.10%	13.50%	15.87%	16.43%	19.02%	20.18%	21.01%
21	12.42%	12.79%	14.98%	15.49%	17.86%	18.91%	19.67%
22	11.81%	12.16%	14.18%	14.65%	16.84%	17.80%	18.50%
23	11.26%	11.59%	13.47%	13.90%	15.93%	16.83%	17.47%
24	10.76%	11.07%	12.82%	13.23%	15.12%	15.96%	16.56%
25	11.34%	11.89%	15.30%	16.14%	20.28%	22.23%	23.68%
26	10.80%	11.30%	14.37%	15.12%	18.78%	20.49%	21.75%
27	10.31%	10.77%	13.56%	14.24%	17.51%	19.01%	20.13%
28	9.87%	10.29%	12.84%	13.45%	16.40%	17.75%	18.74%
29	9.46%	9.85%	12.20%	12.76%	15.44%	16.65%	17.55%
30	9.09%	9.45%	11.62%	12.14%	14.59%	15.70%	16.50%
31	8.75%	9.09%	11.10%	11.58%	13.83%	14.85%	15.59%
32	8.43%	8.75%	10.63%	11.07%	13.16%	14.09%	14.77%
33	8.14%	8.44%	10.19%	10.61%	12.55%	13.42%	14.05%
34	7.87%	8.15%	9.80%	10.19%	12.00%	12.81%	13.39%
35	7.61%	7.88%	9.43%	9.80%	11.50%	12.26%	12.80%
36	7.37%	7.63%	9.10%	9.44%	11.04%	11.75%	12.27%
37	7.87%	8.30%	10.99%	11.66%	14.98%	16.55%	17.72%
38	7.60%	8.00%	10.49%	11.10%	14.11%	15.52%	16.57%
39	7.35%	7.73%	10.03%	10.60%	13.35%	14.63%	15.57%
40	7.12%	7.47%	9.62%	10.14%	12.67%	13.83%	14.69%
41	6.90%	7.23%	9.24%	9.73%	12.06%	13.13%	13.91%
42	6.70%	7.01%	8.89%	9.34%	11.51%	12.49%	13.22%
43	6.50%	6.80%	8.57%	8.99%	11.01%	11.92%	12.59%
44	6.32%	6.60%	8.27%	8.67%	10.55%	11.40%	12.02%
45	6.15%	6.42%	8.00%	8.37%	10.14%	10.93%	11.51%
46	5.99%	6.24%	7.74%	8.09%	9.76%	10.50%	11.04%
47	5.84%	6.08%	7.50%	7.84%	9.41%	10.11%	10.62%
48	5.70%	5.92%	7.28%	7.60%	9.08%	9.74%	10.22%
49	6.12%	6.49%	8.84%	9.43%	12.37%	13.77%	14.82%
50	5.95%	6.30%	8.50%	9.05%	11.76%	13.03%	13.99%
51	5.80%	6.13%	8.19%	8.70%	11.20%	12.37%	13.24%
52	5.65%	5.96%	7.90%	8.38%	10.70%	11.78%	12.58%
53	5.51%	5.80%	7.64%	8.09%	10.25%	11.25%	11.98%
54	5.37%	5.66%	7.39%	7.81%	9.83%	10.76%	11.44%
55	5.24%	5.52%	7.16%	7.56%	9.45%	10.32%	10.95%
56	5.12%	5.38%	6.94%	7.32%	9.10%	9.91%	10.50%
57	5.01%	5.26%	6.74%	7.10%	8.78%	9.54%	10.09%
58	4.90%	5.14%	6.55%	6.89%	8.48%	9.19%	9.71%
59	4.79%	5.02%	6.37%	6.69%	8.20%	8.88%	9.37%
60	4.69%	4.91%	6.20%	6.51%	7.94%	8.58%	9.04%

Gdig = 1.05		10%	12%	22%	24%	32%	35%	37%
	m							
1		91.33%	94.09%	110.77%	114.81%	134.30%	143.34%	150.05%
2		53.92%	55.39%	64.11%	66.19%	76.01%	80.48%	83.75%
3		38.12%	39.11%	44.90%	46.27%	52.69%	55.57%	57.67%
4		29.45%	30.19%	34.50%	35.51%	40.24%	42.35%	43.88%
5		23.98%	24.57%	28.00%	28.80%	32.53%	34.18%	35.38%
6		20.23%	20.71%	23.55%	24.21%	27.28%	28.64%	29.63%
7		17.48%	17.90%	20.32%	20.88%	23.49%	24.65%	25.48%
8		15.39%	15.76%	17.87%	18.36%	20.62%	21.62%	22.35%
9		13.75%	14.07%	15.94%	16.37%	18.38%	19.26%	19.90%
10		12.42%	12.71%	14.39%	14.78%	16.57%	17.36%	17.93%
11		11.33%	11.59%	13.11%	13.46%	15.09%	15.80%	16.32%
12		10.41%	10.65%	12.04%	12.37%	13.85%	14.50%	14.97%
13		10.55%	11.01%	13.79%	14.47%	17.79%	19.34%	20.50%
14		9.70%	10.08%	12.43%	13.00%	15.73%	16.99%	17.92%
15		8.97%	9.31%	11.34%	11.82%	14.13%	15.19%	15.96%
16		8.36%	8.66%	10.43%	10.86%	12.85%	13.76%	14.42%
17		7.82%	8.09%	9.67%	10.04%	11.80%	12.59%	13.17%
18		7.35%	7.59%	9.02%	9.35%	10.92%	11.62%	12.13%
19		6.94%	7.16%	8.45%	8.76%	10.17%	10.80%	11.26%
20		6.57%	6.77%	7.96%	8.24%	9.53%	10.10%	10.51%
21		6.24%	6.43%	7.52%	7.78%	8.96%	9.49%	9.87%
22		5.94%	6.12%	7.13%	7.37%	8.47%	8.95%	9.30%
23		5.67%	5.84%	6.78%	7.00%	8.02%	8.47%	8.80%
24		5.43%	5.58%	6.47%	6.68%	7.63%	8.05%	8.35%
25		5.73%	6.01%	7.74%	8.17%	10.27%	11.26%	12.00%
26		5.46%	5.71%	7.27%	7.66%	9.51%	10.38%	11.02%
27		5.22%	5.45%	6.87%	7.21%	8.87%	9.63%	10.20%
28		4.99%	5.21%	6.50%	6.82%	8.31%	9.00%	9.50%
29		4.79%	4.99%	6.18%	6.47%	7.83%	8.45%	8.90%
30		4.61%	4.79%	5.90%	6.16%	7.40%	7.97%	8.38%
31		4.44%	4.61%	5.63%	5.88%	7.03%	7.54%	7.92%
32		4.28%	4.44%	5.40%	5.62%	6.69%	7.16%	7.51%
33		4.13%	4.28%	5.18%	5.39%	6.38%	6.83%	7.15%
34		4.00%	4.14%	4.98%	5.18%	6.11%	6.52%	6.82%
35		3.87%	4.00%	4.80%	4.99%	5.86%	6.25%	6.53%
36		3.75%	3.88%	4.63%	4.81%	5.63%	5.99%	6.26%
37		4.01%	4.23%	5.62%	5.97%	7.69%	8.50%	9.11%
38		3.87%	4.08%	5.36%	5.68%	7.24%	7.97%	8.51%
39		3.75%	3.94%	5.13%	5.42%	6.84%	7.50%	7.99%
40		3.63%	3.81%	4.92%	5.19%	6.49%	7.09%	7.54%
41		3.52%	3.69%	4.72%	4.97%	6.18%	6.73%	7.13%
42		3.42%	3.58%	4.54%	4.78%	5.89%	6.40%	6.78%
43		3.32%	3.47%	4.38%	4.60%	5.64%	6.11%	6.45%
44		3.23%	3.37%	4.23%	4.44%	5.41%	5.85%	6.16%
45		3.14%	3.28%	4.09%	4.28%	5.19%	5.60%	5.90%
46		3.06%	3.19%	3.96%	4.14%	5.00%	5.38%	5.66%
47		2.98%	3.11%	3.84%	4.01%	4.82%	5.18%	5.45%
48		2.91%	3.03%	3.72%	3.89%	4.66%	5.00%	5.25%
49		3.13%	3.33%	4.55%	4.86%	6.40%	7.14%	7.69%
50		3.05%	3.23%	4.37%	4.66%	6.07%	6.74%	7.24%
51		2.97%	3.14%	4.21%	4.48%	5.78%	6.39%	6.85%
52		2.89%	3.05%	4.06%	4.31%	5.52%	6.08%	6.50%
53		2.82%	2.97%	3.92%	4.16%	5.28%	5.80%	6.18%
54		2.75%	2.90%	3.80%	4.02%	5.07%	5.55%	5.90%
55		2.68%	2.83%	3.68%	3.88%	4.87%	5.32%	5.65%
56		2.62%	2.76%	3.57%	3.76%	4.69%	5.11%	5.41%
57		2.56%	2.69%	3.46%	3.65%	4.52%	4.91%	5.20%
58		2.51%	2.63%	3.36%	3.54%	4.36%	4.74%	5.01%
59		2.46%	2.57%	3.27%	3.44%	4.22%	4.57%	4.83%
60		2.40%	2.52%	3.19%	3.34%	4.09%	4.42%	4.66%

Gdig = 1.01		10%	12%	22%	24%	32%	35%	37%
m								
1		14.18%	14.52%	16.52%	16.98%	19.15%	20.11%	20.80%
2		9.24%	9.45%	10.72%	11.02%	12.38%	12.98%	13.41%
3		6.85%	7.01%	7.93%	8.15%	9.14%	9.58%	9.90%
4		5.44%	5.57%	6.30%	6.47%	7.25%	7.59%	7.84%
5		4.51%	4.62%	5.22%	5.36%	6.00%	6.29%	6.49%
6		3.86%	3.94%	4.46%	4.58%	5.12%	5.36%	5.54%
7		3.37%	3.44%	3.89%	3.99%	4.47%	4.68%	4.83%
8		2.99%	3.05%	3.45%	3.54%	3.96%	4.15%	4.28%
9		2.68%	2.74%	3.10%	3.18%	3.56%	3.72%	3.84%
10		2.44%	2.49%	2.81%	2.89%	3.23%	3.38%	3.49%
11		2.23%	2.28%	2.58%	2.64%	2.96%	3.09%	3.19%
12		2.06%	2.10%	2.38%	2.44%	2.73%	2.85%	2.94%
13		2.09%	2.18%	2.72%	2.85%	3.49%	3.79%	4.01%
14		1.92%	2.00%	2.46%	2.57%	3.10%	3.34%	3.52%
15		1.79%	1.85%	2.25%	2.34%	2.79%	3.00%	3.15%
16		1.67%	1.72%	2.07%	2.16%	2.55%	2.72%	2.85%
17		1.56%	1.61%	1.93%	2.00%	2.35%	2.50%	2.61%
18		1.47%	1.52%	1.80%	1.87%	2.18%	2.32%	2.42%
19		1.39%	1.43%	1.69%	1.75%	2.03%	2.16%	2.25%
20		1.32%	1.36%	1.59%	1.65%	1.91%	2.02%	2.10%
21		1.25%	1.29%	1.51%	1.56%	1.80%	1.90%	1.98%
22		1.19%	1.23%	1.43%	1.48%	1.70%	1.80%	1.87%
23		1.14%	1.17%	1.36%	1.41%	1.61%	1.70%	1.77%
24		1.09%	1.12%	1.30%	1.34%	1.54%	1.62%	1.68%
25		1.16%	1.21%	1.56%	1.65%	2.08%	2.28%	2.43%
26		1.10%	1.15%	1.47%	1.55%	1.92%	2.10%	2.23%
27		1.05%	1.10%	1.39%	1.46%	1.79%	1.95%	2.06%
28		1.01%	1.05%	1.32%	1.38%	1.68%	1.82%	1.92%
29		0.97%	1.01%	1.25%	1.31%	1.58%	1.71%	1.80%
30		0.93%	0.97%	1.19%	1.25%	1.50%	1.61%	1.70%
31		0.90%	0.93%	1.14%	1.19%	1.42%	1.53%	1.60%
32		0.87%	0.90%	1.09%	1.14%	1.36%	1.45%	1.52%
33		0.84%	0.87%	1.05%	1.09%	1.30%	1.39%	1.45%
34		0.81%	0.84%	1.01%	1.05%	1.24%	1.32%	1.38%
35		0.78%	0.81%	0.97%	1.01%	1.19%	1.27%	1.33%
36		0.76%	0.79%	0.94%	0.98%	1.14%	1.22%	1.27%
37		0.81%	0.86%	1.15%	1.22%	1.57%	1.74%	1.87%
38		0.79%	0.83%	1.09%	1.16%	1.48%	1.63%	1.74%
39		0.76%	0.80%	1.04%	1.10%	1.40%	1.53%	1.63%
40		0.74%	0.77%	1.00%	1.06%	1.32%	1.45%	1.54%
41		0.72%	0.75%	0.96%	1.01%	1.26%	1.37%	1.46%
42		0.69%	0.73%	0.93%	0.97%	1.20%	1.31%	1.38%
43		0.67%	0.71%	0.89%	0.94%	1.15%	1.25%	1.32%
44		0.66%	0.69%	0.86%	0.90%	1.10%	1.19%	1.26%
45		0.64%	0.67%	0.83%	0.87%	1.06%	1.14%	1.21%
46		0.62%	0.65%	0.81%	0.84%	1.02%	1.10%	1.16%
47		0.61%	0.63%	0.78%	0.82%	0.98%	1.06%	1.11%
48		0.59%	0.62%	0.76%	0.79%	0.95%	1.02%	1.07%
49		0.64%	0.68%	0.93%	1.00%	1.32%	1.47%	1.59%
50		0.62%	0.66%	0.90%	0.96%	1.25%	1.39%	1.49%
51		0.60%	0.64%	0.86%	0.92%	1.19%	1.31%	1.41%
52		0.59%	0.62%	0.83%	0.88%	1.13%	1.25%	1.34%
53		0.57%	0.61%	0.80%	0.85%	1.08%	1.19%	1.27%
54		0.56%	0.59%	0.78%	0.82%	1.04%	1.14%	1.21%
55		0.55%	0.58%	0.75%	0.79%	1.00%	1.09%	1.16%
56		0.54%	0.56%	0.73%	0.77%	0.96%	1.05%	1.11%
57		0.52%	0.55%	0.71%	0.75%	0.93%	1.01%	1.07%
58		0.51%	0.54%	0.69%	0.72%	0.89%	0.97%	1.03%
59		0.50%	0.53%	0.67%	0.70%	0.86%	0.94%	0.99%
60		0.49%	0.51%	0.65%	0.68%	0.84%	0.91%	0.96%

Equivalent APY Conclusion:

Based on current traditional savings account APY values (near 2.25%), The first three Digs tiers are advantageous to contribute to for all values of m and for all tax brackets. Tier four only becomes advantageous to contribute to in the last 10 to 18 months before purchasing a home, depending on the tax bracket.

One limitation of this analysis is that investing is assumed to begin in January. However, shifting the assumption to another month should have little effect on the results of this analysis. Another limitation of this analysis is that it assumes a constant traditional savings account APY for the entire investing duration, but the APY could increase or decrease. A decrease in the APY of a traditional savings account after already beginning the investing duration into the Digs savings account will not cause any losses. An increase in the APY of a traditional savings account after already beginning the investing duration into the Digs savings account may cause a loss of maximum earnings if continuing to contribute to the same tiers. In this case, which tiers to contribute to should be reevaluated.

This analysis is limited to a comparison of the Digs savings account to a traditional savings account. The Digs savings account can also be compared to something with a different rate of return, such as investing in stocks or bonds. Although these alternatives to traditional savings accounts may yield larger returns, there is more risk involved. Additionally, long-term capital gains would apply here if the total investing time exceeds 12 months, and therefore the equivalent APY tables created would need to be adjusted by using Equation (6b) for any value of m instead of using Equation (6c), since the principal grows tax-free until the money is withdrawn.

Equivalent APY Script:

Below is the Python script used to solve for the equivalent APYs:

```
#!/usr/bin/python3
from scipy import optimize
import math

def _Gm(r,C,m):
    Gm = 0
    if m != 0:
        x=12*math.floor((m-1)/12)
        Gx = _Gm(r,C=C,m=x)
        for i in range(1,m-x+1):
            Gm += (1+r/12)**i
        Gm += x*Gx*(1+r/12)**(m-x)
        Gm = C*(Gm/m - 1) + 1
    return Gm

def Gm(r,C,m,Gd):
    eqn = _Gm(r,C=C,m=m)
    eqn -= Gd
    return eqn

lim = 60
Gdigs = 1.01
C_list = [0.9,0.88,0.78,0.76,0.68,0.65,0.63] #C=(1-t)
months_list = [i for i in range(1,lim+1)]
results = {}
C_table = ["{}%".format(str(round((1-i)*100))) for i in C_list]
print("Gdigs={}".format(Gdigs))
print("m,{}".format(",".join(C_table)))
```

```

for months in months_list:
    results[months] = []
    for C in C_list:
        def funct(r,C=C,m=months,Gd=Gdigs):
            return Gm(r,C,m,Gd)
        r = optimize.root(funct, [1]).x[0]
        APY = ((r/12+1)**12-1) # print as proportion
        results[months].append(str(APY))
for months in results:
    print("{} , {}".format(months, ",".join(results[months])))

```

Mortgage APR Analysis:

The equation for monthly mortgage payment is shown below:

$$M = P \frac{r(1+r)^n}{(1+r)^n - 1} \quad (8)$$

where:

M=monthly mortgage payment

P=principal mortgage amount

r=monthly interest rate

n=mortgage duration in months

Equation (8) can be manipulated to determine the total mortgage interest for the life of the mortgage, shown below:

$$I = Mn - P = P \left(\frac{nr(1+r)^n}{(1+r)^n - 1} - 1 \right) \quad (9)$$

The earned funds from the Digs savings account are only honored by a specific network of lenders (see <https://digs.co/lenders/>). So another determining factor in using the Digs savings account would be comparing the mortgage rates of these in-network lenders to best mortgage rates available. If the increase in owed interest on a mortgage due to using an in-network lender is greater than the money earned from the Digs savings account, then the Digs savings account is not worth using. The analysis here will be bound by assuming that the traditional savings account alternative earns no interests. Therefore, the maximum amount that can be earned from the Digs savings account is \$2,340 (five years), including the \$1/month fee:

$$I_{digs} - I_{trad} = 2340 \quad (10)$$

The below equation converts the monthly interest rate to an APR, which is what mortgage rates are represented as:

$$APR = 12r \quad (11)$$

Using Equations (9), (10), and (11), the value of APR_{trad} can be determined for various values for P, APR_{digs} , and n, where APR_{trad} represents the mortgage rate that would cause the Digs savings account to

become unprofitable, and where APR_{digs} represents the best available lender in the Digs savings account network of lenders.

For a 30 year fixed mortgage ($n=360$), APR_{trad} values have been determined for principal mortgage amounts between \$50,000 and \$500,000, and for APR_{digs} values between 1% and 10%. The following page can be used to find how much lower of an APR would be necessary to cause contributions to the Digs savings account to be unprofitable.

Mortgage APR Conclusion:

In order for the Digs savings account to be profitable, there would need to be no out-of-network mortgage rates available that are less than the mortgage rate in the table of results, for a specified in-network mortgage rate and principal mortgage amount. Since most of the differences between in-network mortgage rates and out-of-network mortgage rates are small, it is unlikely that using the Digs savings account will be profitable.

This analysis assumes that the maximum amount possible is earned from the Digs savings account. Contributing less than the five year maximum or not contributing the maximum to all four Digs tiers will decrease the total amount earned from the Digs savings account from the \$2,340 maximum. Additionally, this analysis neglects considering a traditional savings account, which may make contributing to all four Digs tiers unprofitable (see Equivalent APY Analysis). Contributing to less than four of the Digs tiers will also decrease the total amount earned from the Digs savings account from the \$2,340 maximum. When the total amount earned from the Digs savings account decreases, the difference between in-network mortgages and out-of-network mortgages decreases, which makes contributing to the Digs savings account less likely to be profitable.

Mortgage APR Script:

Below is the Python script used to solve for the out-of-network mortgage rates that cause contributions to the Digs savings account to be unprofitable:

```
#!/usr/bin/python3
from scipy import optimize

n = 30*12
APR_d_inc = 0.05
APR_d_start = 1
APR_d_end = 10
P_inc = 25000
P_start = 50000
P_end = 500000

APR_d_list = [round(i*APR_d_inc,8) for i in
range(int(APR_d_start/APR_d_inc),int(APR_d_end/APR_d_inc)+1)]
P_list = [round(i*P_inc,8) for i in range(int(P_start/P_inc),int(P_end/P_inc)+1)]
results = {}
P_table = [{"{}k".format(str(int(i/1000))) for i in P_list]
print("APR,{}".format(",".join(P_table)))
for APR_d in APR_d_list:
    results[APR_d] = []
    r_d = APR_d/100/12
    for P in P_list:
        def funct(r_t,r_d=r_d,n=n,P=P):
            eqn = r_d*(1+r_d)**n/((1+r_d)**n-1) - r_t*(1+r_t)**n/((1+r_t)**n-1) - 2340/P/n
            return eqn
        r_t = optimize.root(funct, [r_d]).x[0]
        APR_t = r_t*100*12
        results[APR_d].append(str(APR_t))
for APR_d in results:
    print("{} , {}".format(APR_d, ",".join(results[APR_d])))
```

Final Thoughts:

From the second analysis, a combination of a smaller in-network mortgage rate, a smaller principal mortgage amount, and a longer contribution duration to the Digs savings account will make it more

likely that contributing to the Digs savings account will be profitable. Further analysis can be performed by determining which tiers to contribute to based on the first analysis, and adjusting the table in the second analysis for the total amount earned from the Digs savings account, which will likely be less than \$2,340, due to less months spent contributing and/or not contributing to all four tiers. After the table is adjusted, the lowest in-network and lowest out-of-network mortgage rates can be determined and compared to the adjusted table. Another strategy would be to diversify contributions since mortgage rates, mortgage values, and other factors can change during the investment period. By contributing to the first tier only, losses are minimized for both profitable and unprofitable outcomes. Using the Digs savings account will also allow access to additional features of the account, such as expert content & advice, home buyers checklist, and lender recommendations (see <https://digs.co/features/>).